

CA-Final AFM Test 4

Topics Covered: Portfolio Management

Max Marks: 40

Time: 1 Hour

Question 1(10 Marks):

Following are the details of a portfolio consisting of three shares:

Share	Portfolio weight	Beta	Expected return in %	Total variance
D	0.20	0.40	14	0.015
E	0.50	0.50	15	0.025
F	0.30	1.10	21	0.100

Standard Deviation of Market Portfolio Returns = 10% You are

given the following additional data:

Covariance (D, E) = 0.030

Covariance (D, F) = 0.020

Covariance (E, F) = 0.040

Calculate the Portfolio variance considering:

(i) Correlation between each pair of securities.

(ii) Co-movement between securities due to change in the market index.

Answer:

(i) We can compute the Portfolio variance on the basis of Correlation between each pair of securities as follows:

$$\begin{aligned}
 &= (w_A \times w_A \times \sigma_A^2) + (w_A \times w_B \times \text{Cov}_{AB}) + (w_A \times w_C \times \text{Cov}_{AC}) + (w_B \times w_A \times \text{Cov}_{AB}) + (w_B \times w_B \times \sigma_B^2) + \\
 &\quad (w_B \times w_C \times \text{Cov}_{BC}) + (w_C \times w_A \times \text{Cov}_{CA}) + (w_C \times w_B \times \text{Cov}_{CB}) + (w_C \times w_C \times \sigma_C^2) \\
 &= (0.20 \times 0.20 \times 0.015) + (0.20 \times 0.50 \times 0.030) + (0.20 \times 0.30 \times 0.020) + (0.20 \times 0.50 \times 0.030) \\
 &\quad + (0.50 \times 0.50 \times 0.025) + (0.50 \times 0.30 \times 0.040) + (0.30 \times 0.20 \times 0.020) + (0.30 \times 0.50 \times 0.040) \\
 &\quad + (0.30 \times 0.30 \times 0.10) \\
 &= 0.0006 + 0.0030 + 0.0012 + 0.0030 + 0.00625 + 0.0060 + 0.0012 + 0.0060 + 0.0090 \\
 &= 0.0363
 \end{aligned}$$

(ii) To compute the Portfolio variance considering the Co-movement between securities due to change in the market index first we shall compute the Beta of the portfolio is as follows:

$$0.20 \times 0.40 + 0.50 \times 0.50 + 0.30 \times 1.10 = 0.66$$

Then we shall compute the Residual Variance by separating the Systematic Risk from total risk as follows:

Systematic Risk of each security shall be computed as follows:

$$\beta_A^2 \times \sigma_M^2 = (0.40)^2(0.01) = 0.0016$$

$$\beta_B^2 \times \sigma_M^2 = (0.50)^2(0.01) = 0.0025$$

$$\beta_C^2 \times \sigma_M^2 = (1.10)^2(0.01) = 0.0121$$

The Residual Variance of each security

$$A \quad 0.015 - 0.0016 = 0.0134$$

$$B \quad 0.025 - 0.0025 = 0.0225$$

$$C \quad 0.100 - 0.0121 = 0.0879$$

Then Portfolio variance shall be computed using Sharpe Index Model as follows: Systematic

$$\text{Variance of Portfolio} = (0.10)^2 \times (0.66)^2 = 0.004356$$

$$\begin{aligned}\text{Unsystematic Variance of Portfolio} &= 0.0134 \times (0.20)^2 + 0.0225 \times (0.50)^2 + 0.0879 \times (0.30)^2 \\ &= 0.014072\end{aligned}$$

$$\text{Total Variance} = 0.004356 + 0.014072 = 0.018428$$

Question 2(8 Marks): M is interested to construct a Portfolio of Securities A and B. He has collected the following information:

	A	B
Expected Return (ER)	19%	23%
Risk (σ)	14%	18%

M has 4 Portfolio options of A and B as follows:

- (i) 50% of funds in each A and B
- (ii) 75% of funds in A and 25% in B
- (iii) 25% of funds in A and 75% in B
- (iv) 60% of funds in A and 40% in B

Co-efficient of correlation (r) between A and B is 0.16. You are required to calculate:

- (i) Expected Return under different Portfolio Options.
- (ii) Risk Factor associated with these Portfolio Options.
- (iii) Which Portfolio is best from the point of view of Risk?
- (iv) Which Portfolio is best from the point of view of Return?

Answer:

Two asset portfolio

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12} \text{ Or}$$

$$\sigma_p = \sqrt{w_1\sigma_1 + w_2\sigma_2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}} \text{ Substituting}$$

the respective values we get,

- (i) 50% of funds in each of A and B

$$E_p = 0.50 \times 19\% + 0.50 \times 23\% = 21\%$$

$$\sigma_p^2 = (0.50)^2(14\%)^2 + (0.50)^2(18\%)^2 + 2(0.50)(0.50)(0.16)(14\%)(18\%)$$

$$\sigma_p^2 = 49 + 81 + 20.16 = 150.16$$

$$\sigma_p = 12.25\%$$

- (ii) 75% in A and 25% in B

$$E_p = 0.75 \times 19\% + 0.25 \times 23\% = 20\%$$

$$\sigma_p^2 = (0.75)^2(14\%)^2 + (0.25)^2(18\%)^2 + 2(0.75)(0.25)(0.16)(14\%)(18\%)$$

$$\sigma_p^2 = 110.25 + 20.25 + 15.12 = 145.62$$

$$\sigma_p = 12.07\%$$

- (iii) 25% in A and 75% in B

$$E_p = 0.25 \times 19\% + 0.75 \times 23\% = 22\%$$

$$\sigma_p^2 = (0.25)^2(14\%)^2 + (0.75)^2(18\%)^2 + 2(0.25)(0.75)(0.16)(14\%)(18\%)$$

$$\sigma_p^2 = 12.25 + 182.25 + 15.12 = 209.62$$

$$\sigma_p = 14.48\%$$

- (iv) 60% in A and 40% in B

$$E_p = 0.60 \times 19\% + 0.40 \times 23\% = 20.6\%$$

$$\sigma_p^2 = (0.60)^2(14\%)^2 + (0.40)^2(18\%)^2 + 2(0.60)(0.40)(0.16)(14\%)(18\%)$$

$$\sigma_p^2 = 70.56 + 51.84 + 19.35 = 141.75$$

$$\sigma_p = 11.91\%$$

Portfolio	(i)	(ii)	(iii)	(iv)
Return	21	20	22	20.6
σ	12.25	12.07	14.48	11.91

In the terms of return, we see that portfolio (iii) is the best portfolio. In terms of risk we see that portfolio (iv) is the best portfolio.

Question 3(8 Marks): Equity of ABC Ltd. (ABCL) is Rs 500 Crores, its debt, is worth Rs 290 Crores. Printer Division segments value is attributable to 64%, which has an Asset Beta (β_p) of 1.55, balance value is applied on Spares and Consumables Division, which has an Asset Beta (β_{sc}) of 1.40 ABCL Debt beta (β_D) is 0.28.

You are required to calculate:

(i) Equity Beta (β_E),

(ii) Ascertain Equity Beta (β_E), if ABC Ltd. decides to change its Debt Equity position by raising further debt and buying back of equity to have its Debt to Equity Ratio at 1.50.

Assume that the present Debt Beta (β_{D1}) is 0.45 and any further funds raised by way of Debt will have a Beta (β_{D2}) of 0.50.

(iii) Whether the new Equity Beta (β_E) justifies increase in the value of equity on account of leverage?

Answer:

1. Equity Beta

To calculate Equity Beta first we shall calculate Weighted Average of Asset Beta as follows:

$$= 1.55 \times 0.64 + 1.40 \times 0.36$$

$$= 0.992 + 0.504 = 1.496$$

Now we shall compute Equity Beta using the following formula:

$$B_{asset} = B_e \cdot W_e + B_d \cdot W_d$$

$$1.496 = B_e \cdot 500/790 + 0.28 \cdot 290/790$$

$$B_e = 2.20$$

(ii) Equity Beta on change in Capital Structure

Amount of Debt to be raised:

Particulars	Value (in Rs Crore)
Total Value of Firm (Equity Rs 500 crore + Debt Rs 290 crore)	790
Desired Debt Equity Ratio	1.50 : 1.00
Desired Debt Level = $\frac{\text{Total Value} \times \text{Debt Ratio}}{\text{Debt Ratio} + \text{Equity Ratio}}$	474
Less: Value of Existing Debt	(290)
Value of Debt to be Raised	184

$$\text{Equity after Repurchase} = \text{Total value of Firm} - \text{Desired Debt Value}$$

$$= \text{Rs } 790 \text{ Crore} - \text{Rs } 474 \text{ Crore}$$

$$= \text{Rs } 316 \text{ Crore}$$

Weighted Average Beta of ABCL:

Source of Finance	Investment (in Rs Crore)	Weight	Beta of the Division	Weighted Beta
Equity	316	0.4	$\beta_{(E=X)}$	0.4x
Debt - 1	290	0.367	0.45	0.165
Debt - 2	184	0.233	0.50	0.117
	790	Weighted Average Beta		0.282 + (0.4x)

$$\beta_{ABCL} = 0.282 + 0.4x$$

$$1.496 = 0.282 + 0.4x$$

$$0.4x = 1.496 - 0.282$$

$$X = 1.214/0.4 = 3.035$$

$$\beta_{\text{New Equity}} = 3.035$$

Yes, it justifies the increase as it leads to increase in the Value of Equity due to increase in Beta.

Question 4(6 Marks): Ankit has a fund of Rs 10 lacs which he wants to invest in share market with rebalancing target after every 15 days to start with for a period of one month from now. The present NIFTY is 19679. The minimum NIFTY within a month can at most be 18104.68. He wants to know as to how he should rebalance his portfolio under the following situations, according to the theory of Constant Proportion Portfolio Insurance Policy, using "2" as the multiplier:

(i) Immediately to start with.

(ii) 15 days later-being the 1st day of rebalancing if NIFTY falls to 19088.63.

(iii) 15 days further from the above date if the NIFTY touches 20997.493.

Note: Assume that the value of his equity component will change in tandem with that of the NIFTY.

$$\text{Answer: Maximum decline in one month} = \frac{19679 - 18104.68}{19679} \times 100 = 8\%$$

(1) Immediately to start with

$$\begin{aligned} \text{Investment in equity} &= \text{Multiplier} \times (\text{Portfolio value} - \text{Floor value}) \\ &= 2 (10,00,000 - 9,20,000) = \text{Rs } 1,60,000 \end{aligned}$$

Ankit may invest Rs 1,60,000 in equity and balance in risk free securities.

(2) After 15 days

$$\text{Value of equity} = 1,60,000 \times 19088.63 / 19679 = \text{Rs } 1,55,200$$

$$\text{Value of risk free investment} = \text{Rs } 8,40,000$$

$$\text{Total value of portfolio} = \text{Rs } 9,95,200$$

$$\begin{aligned} \text{Investment in equity} &= \text{Multiplier} \times (\text{Portfolio value} - \text{Floor value}) \\ &= 2 (9,95,200 - 9,20,000) = \text{Rs } 1,50,400 \end{aligned}$$

Revised Portfolio:

$$\text{Equity} = \text{Rs } 1,50,400$$

$$\begin{aligned} \text{Risk free Securities} &= \text{Rs } 9,95,200 - \text{Rs } 1,50,400 &= \text{Rs } 8,44,800 \\ (3) \text{ After another 15 days} & & \\ \text{Value of equity} &= 1,50,400 \times 20997.493 / 19088.63 &= \text{Rs } 1,65,440 \\ \text{Value of risk free investment} &= &= \text{Rs } 8,44,800 \\ \text{Total value of portfolio} &= &= \text{Rs } 10,10,240 \\ \text{Investment in equity} &= \text{Multiplier} \times (\text{Portfolio value} - \text{Floor value}) \\ &= 2 (10,10,240 - 9,20,000) &= \text{Rs } 1,80,480 \\ \text{Revised Portfolio:} & & \\ \text{Equity} &= &= \text{Rs } 1,80,480 \\ \\ \text{Risk Free Securities} &= \text{Rs } 10,10,240 - \text{Rs } 1,80,480 &= \text{Rs } 8,29,760 \end{aligned}$$

Ankit should off-load Rs 15,040 of risk free securities and divert to Equity.

Question 5(8 Marks): The returns and market portfolio for a period of four years are as under:

Year	% Return of Stock	% Return on Market
1	10	8
2	12	10
3	9	9
4	3	-1

For stock B, you are required to determine:

- (a) characteristic line; and
- (b) The Systematic and Unsystematic Risk

Answer:

Characteristic line is given by

$$\alpha_i + \beta_i R_m$$

$$\beta_i = (\sum xy - n\bar{x}\bar{y}) / (\sum x^2 - n(\bar{x})^2)$$

$$\alpha_i = \bar{y} - \beta\bar{x}$$

Return on B(Y)	Return on Market (X)	XY	X ²	(x- $\bar{x}$)	(x- $\bar{x}$) ²	(y- $\bar{y}$)	(y- $\bar{y}$) ²
10	8	80	64	1.50	2.25	1.50	2.25
12	10	120	100	3.50	12.25	3.50	12.25
9	9	81	81	2.50	6.25	0.50	0.25
<u>3</u>	<u>-1</u>	<u>-3</u>	<u>1</u>	-7.50	<u>56.25</u>	-5.50	<u>30.25</u>
34	26	278	246		77.00		45.00

$$= \bar{y} = 34 / 4 = 8.50$$

$$= \bar{x} = 26 / 4 = 6.50$$

$$\beta = (\sum xy - n\bar{x}\bar{y}) / (\sum x^2 - n(\bar{x})^2) = (278 - 4(6.50)(8.50)) / (246 - 4(6.50)^2) = 57 / 77 = 0.74$$

$$\alpha = \bar{y} - \beta\bar{x} = 8.50 - 0.74(6.50) = 3.69$$

Hence the characteristic line is $3.69 + 0.74 (R_m)$

$$\text{Total risk of market} = \sigma_m^2 = \sum (x - \bar{x})^2 / n = 77 / 4 = 19.25\%$$

$$\text{Total Risk of stock} = 45 / 4 = 11.25\%$$

$$\text{Systematic Risk} = \beta^2 \sigma_m^2 = (0.74)^2 \times 19.25 = 10.54(\%)$$

Unsystematic Risk is = Total Risk – Systematic Risk
= 11.25 – 10.54 = 0.71(%)